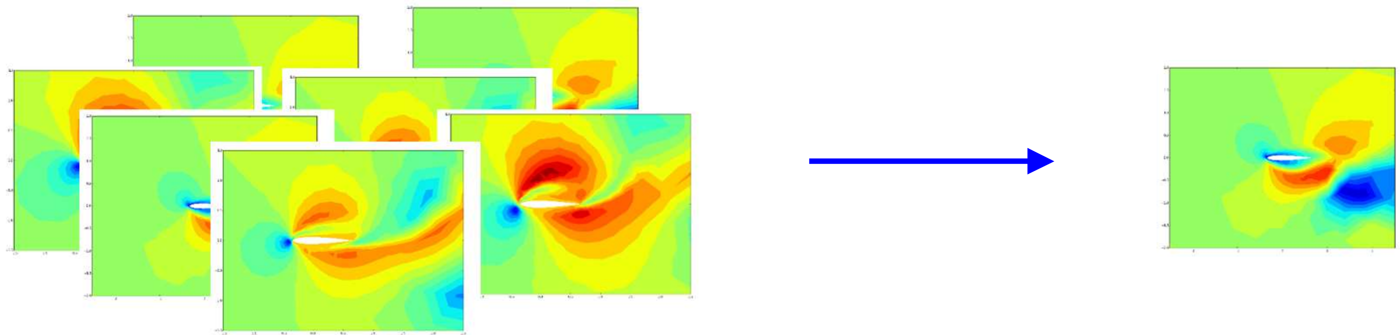
A SPATIAL CLUSTERING ALGORITHM FOR CONSTRUCTING LOCAL REDUCED-ORDER BASES FOR NONLINEAR MODEL REDUCTION



Tina White[2], Charbel Farhat[1,2,3] and P. Avery [1]

[1] Department of Aeronautics and Astronautics

[2]Department of Mechanical Engineering

[3] Institute for Computational and Mathematical Engineering

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NONLINEAR MODEL REDUCTION

- Highly nonlinear High-Dimensional computational Models (HDMs) for dynamical systems
  - Computational Fluid Dynamics (CFD)
  - Computational Structural Dynamics (CSD)
  - Computational physics or partial differential equation-based computational models in general

- Characteristics
  - feature different physical regimes
  - feature moving features (discontinuities, fronts, shocks, …)
  - feature multiple spatial and temporal scales
  - their trajectories can explore many regions of the state-space

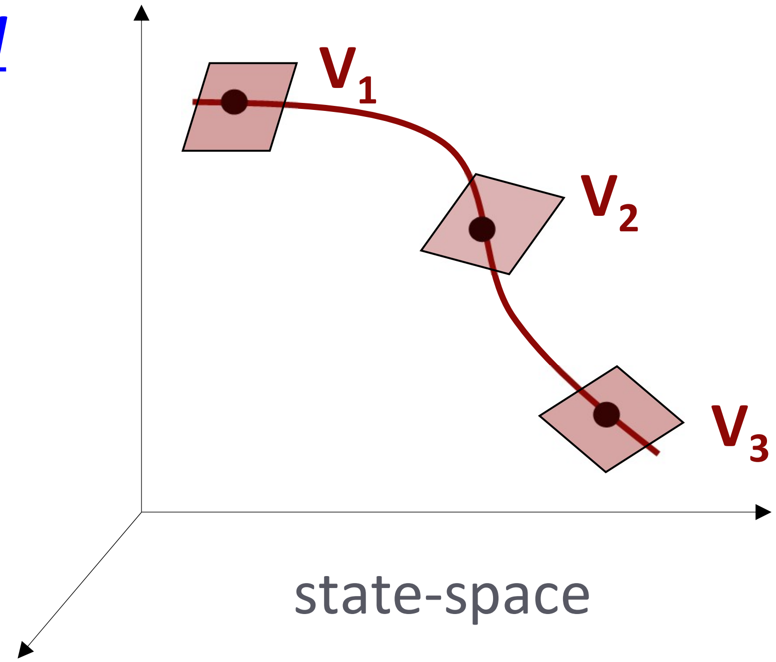
→ their reduction using a global Reduced-Order Basis (ROB) is either inefficient or unfeasible

LOCAL ROBs

- Approximation in a lower-dimensional subspace generated by local ROBs [Amsallem, Zahr & Farhat, 2012]

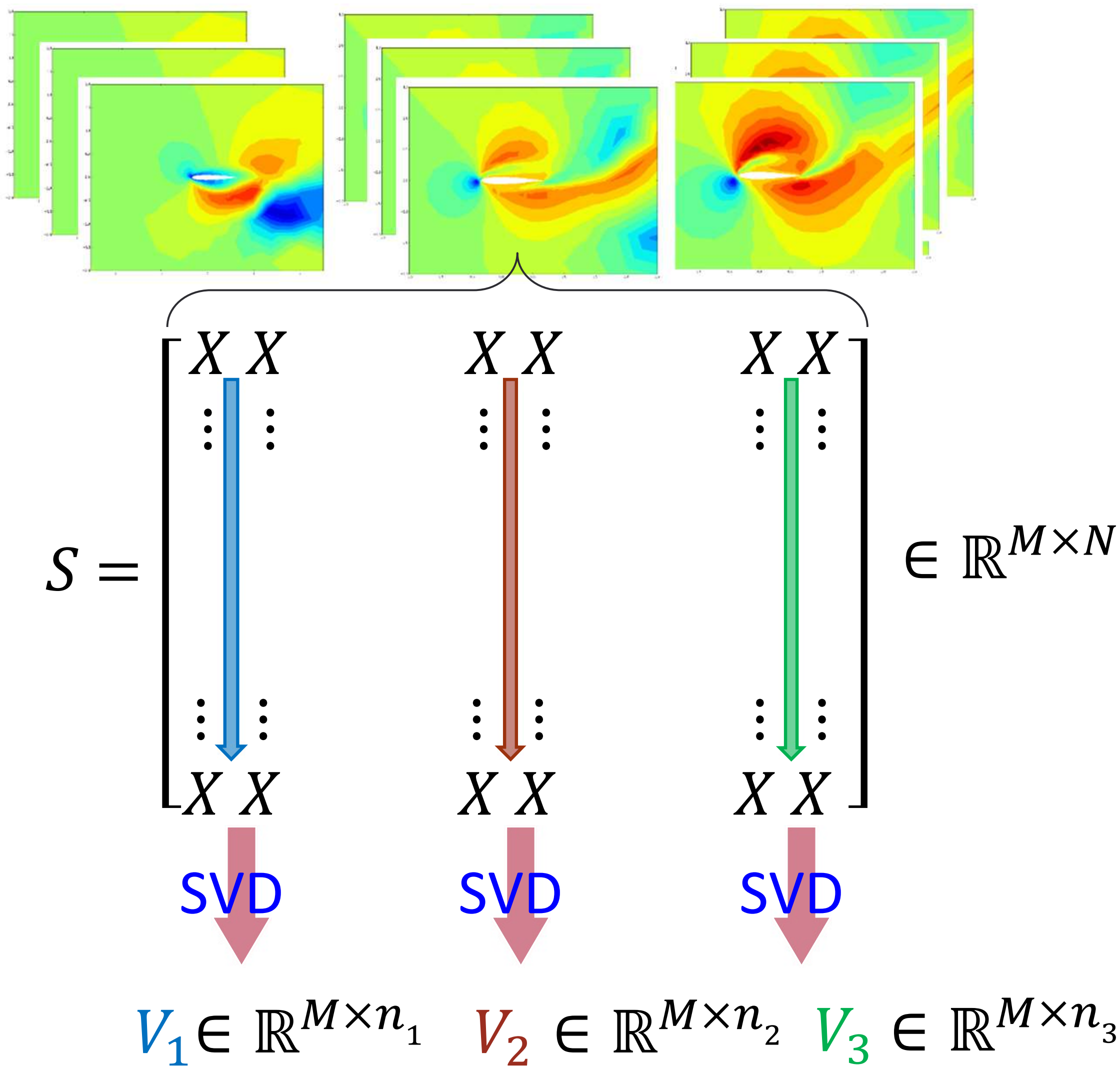
  - locality on the ***solution manifold*** (not necessarily space or time)
  - local ROBs tailored to distinct physical regimes

"column" clustering of the matrix of solution snapshots



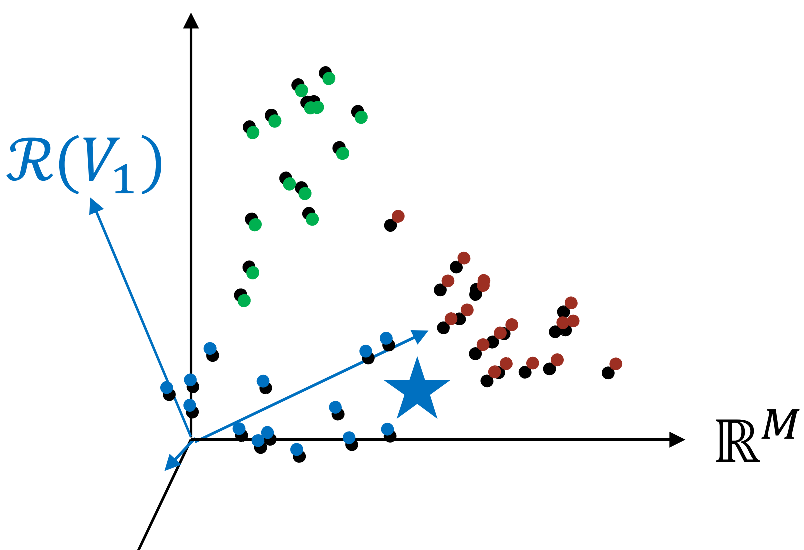
COLUMN CLUSTERING

- Column clustering using, for example, the k-means algorithm



COLUMN CLUSTERING

- Benefits of column clustering
  - groups similar states together ➡ identifies ***distinct regimes***
  - ***reduces dimensionality*** in each cluster of the solution manifold.



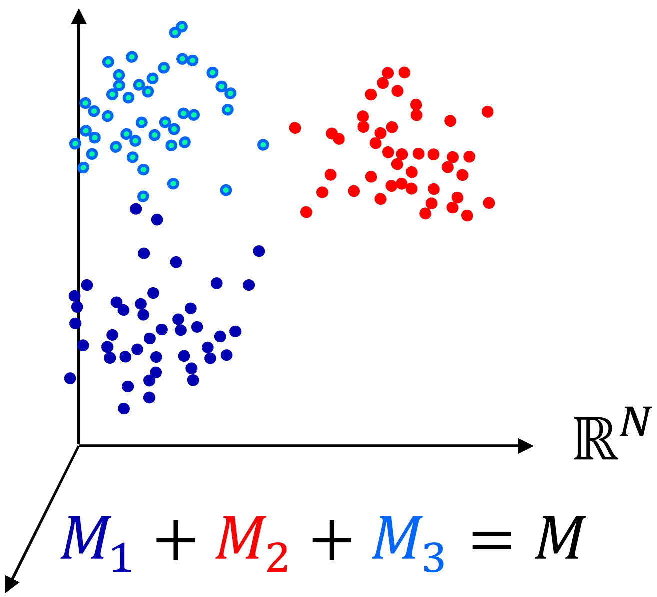
$$n_1 + n_2 + n_3 \leq N$$

$$w \sim V_1 y$$
$$w \sim V_2 y$$
$$w \sim V_3 y$$

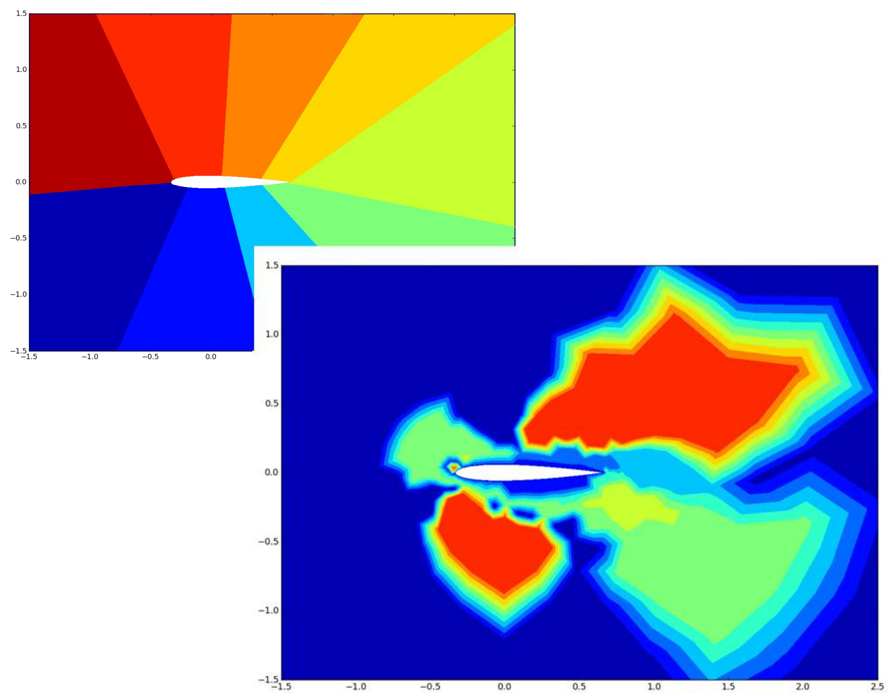
$$\boxed{\begin{array}{l} n_1 \ll M \\ n_2 \ll M \\ n_3 \ll M \end{array}}$$

SPATIAL/ROW CLUSTERING

- Spatial (or “row”) clustering: basic idea

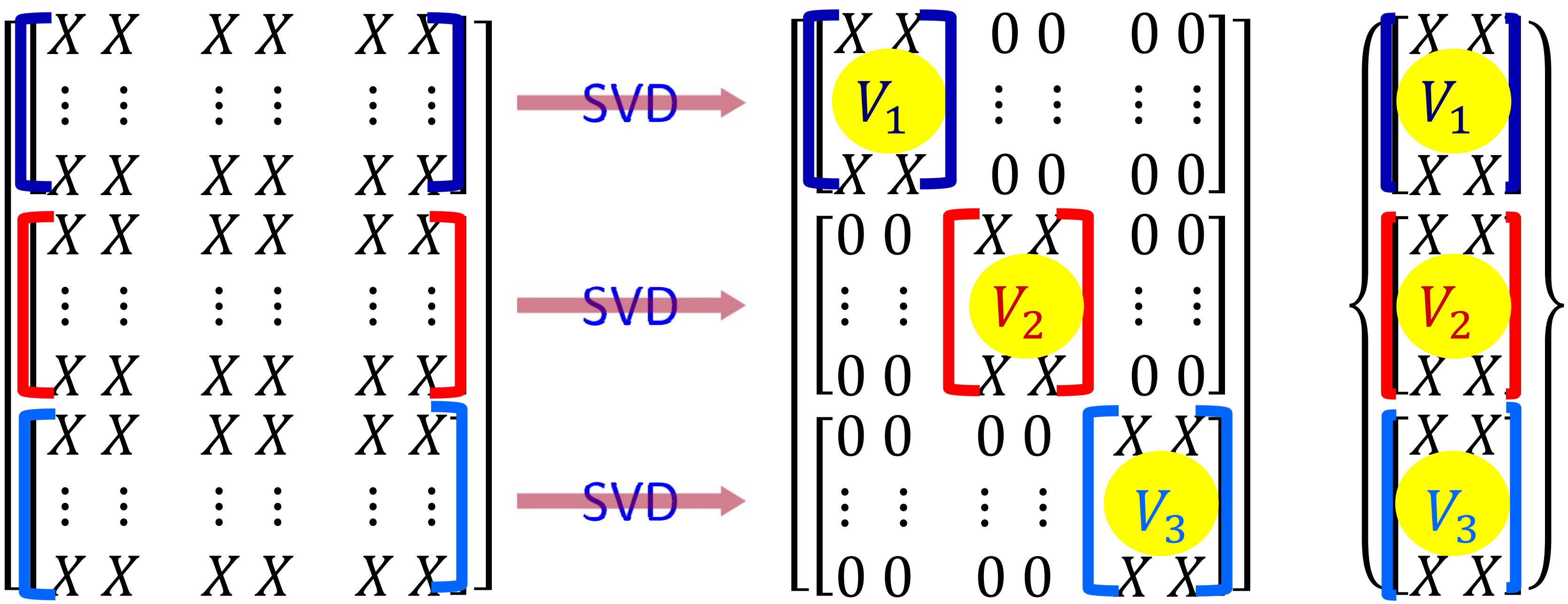


$$M_1 + M_2 + M_3 = M$$



SPATIAL/ROW CLUSTERING

- Spatial (or “row”) clustering: sparsification property



$S \in \mathbb{R}^{M\times N}$

sparsified ROB based on row clustering (local modes)

compact storage

EXPECTED BENEFITS

- Locality in the physical space

  - improved accuracy/efficiency for local/localized phenomena (i.e., steady-state shocks)

- Sparsification property

  - reduced online cost for a given ROB size and therefore for a given accuracy (scenario 1)

    i.e.: $n_1 = n_2 = n_3 = n_g/3 \Rightarrow n_1 + n_2 + n_3 = n_g$

  - increased ROB size and therefore increased accuracy for a given online cost (scenario 2)

    i.e.: $n_1 = n_2 = n_3 = n_g \Rightarrow n_1 + n_2 + n_3 = 3n_g$

SPATIAL/ROW CLUSTERING

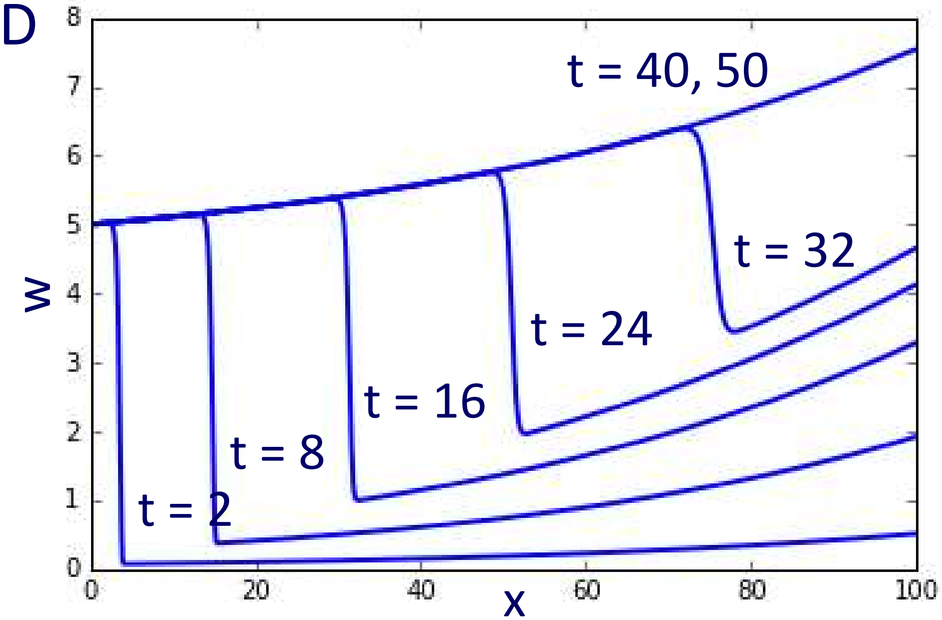
- Spatial (or "row") clustering: relationship to column clustering
  - an alternative approach to "column" clustering
    - locality in the physical space instead of the solution manifold ➡ identifies ***local/localization phenomena*** such as fronts, contacts, shocks, discontinuities, and shear bands, to name a few
  - a complementary approach to "column" clustering
    - ***reduces computational complexity*** by sparsifying a ROB instead of reducing dimensionality
    - can be combined with column clustering to ***reduce both dimensionality and computational complexity***

APPLICATION: SHOCK PROPAGATION

- Inviscid Burgers' problem in 1D

$$\frac{\partial U}{\partial t} + \frac{1}{2}\frac{\partial U^2}{\partial x} = 0$$

$$U(0,t) = 3, \quad U(x,0) = 1$$

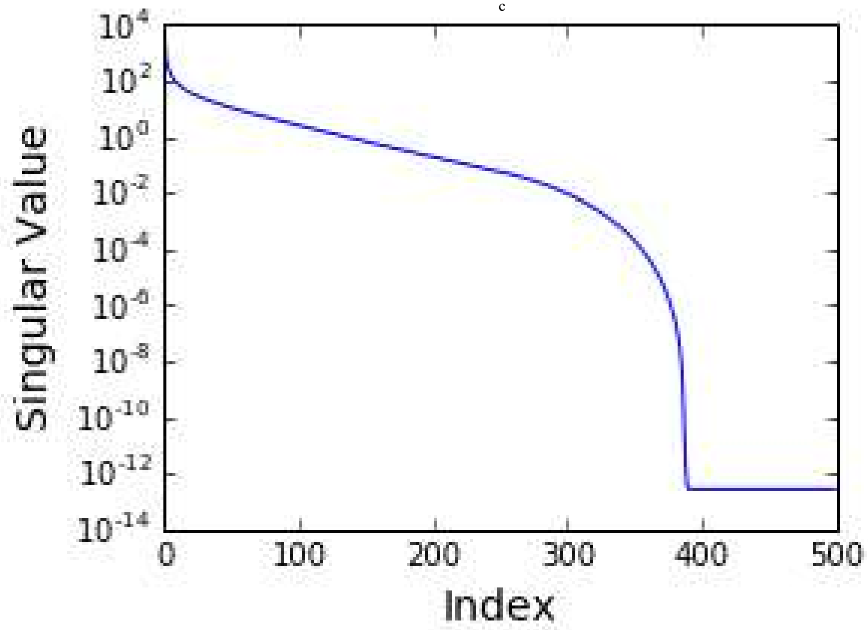


- Semi-discretization by the finite volume method
  - Godunov's scheme ($M = 1000$) with 501 time steps
- Moving shock ➡ expected benefits of spatial clustering are those due to the sparsification property
- Accuracy metric: RMS error of the reduced model solution

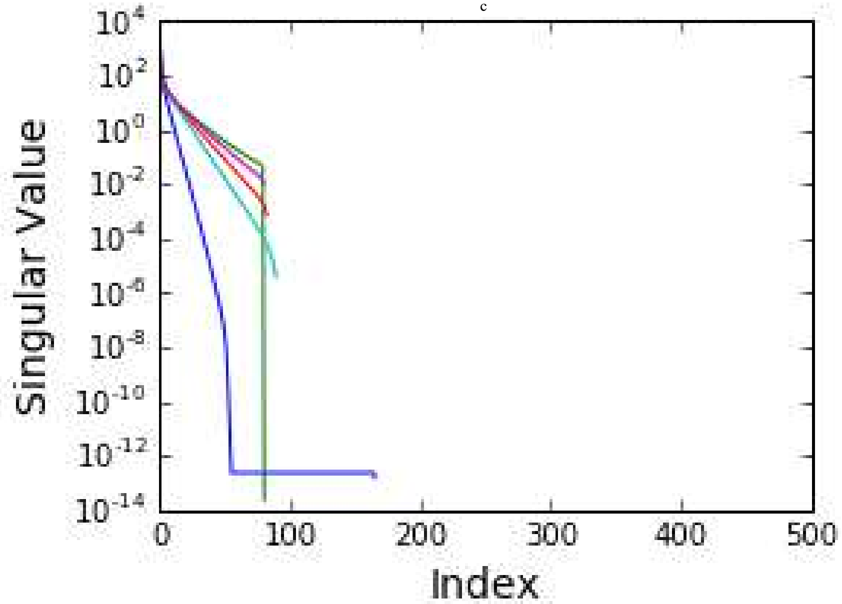
APPLICATION: SHOCK PROPAGATION

- Inviscid Burger problem in 1D

500 snapshots, 1 cluster



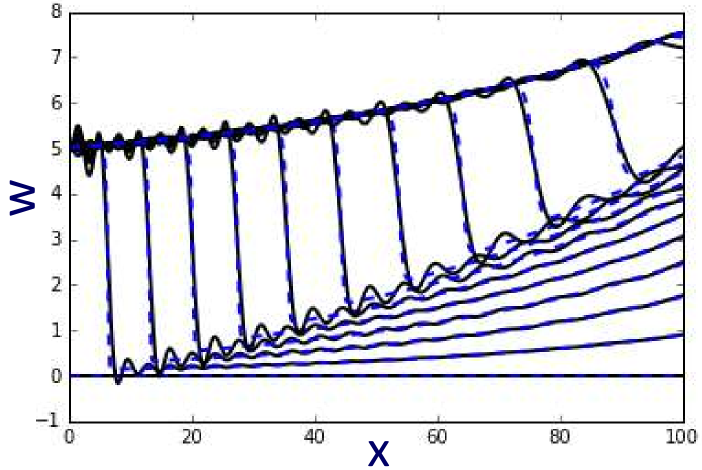
500 snapshots
5 column clusters



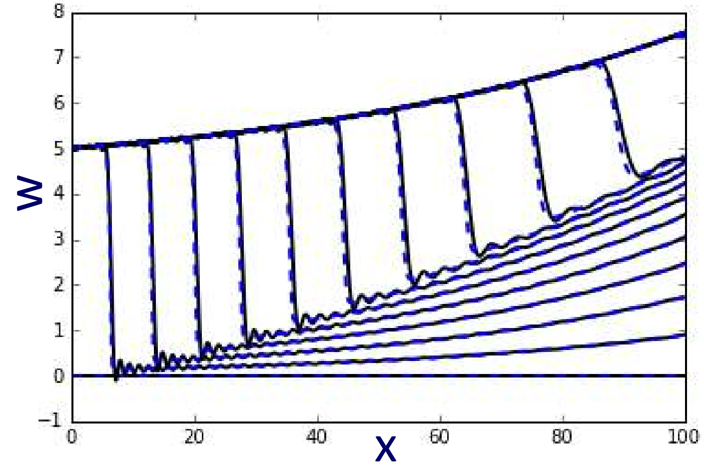
MOST RELEVANT CHARACTERISTIC

- Online accuracy vs retained energy content of the singular values

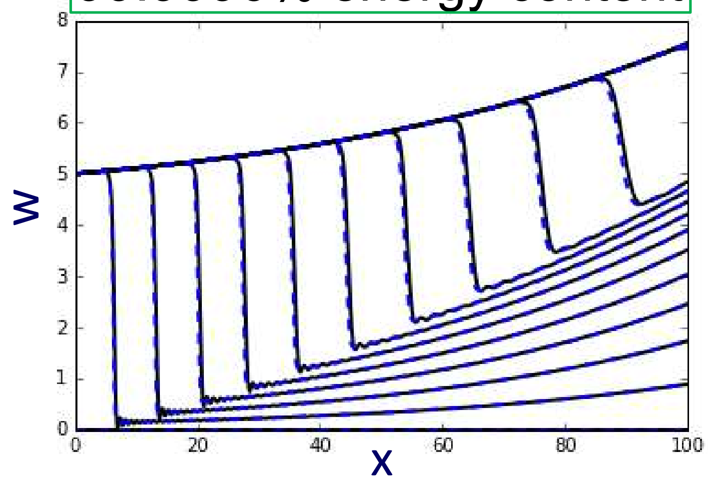
99.96% energy content



99.996% energy content

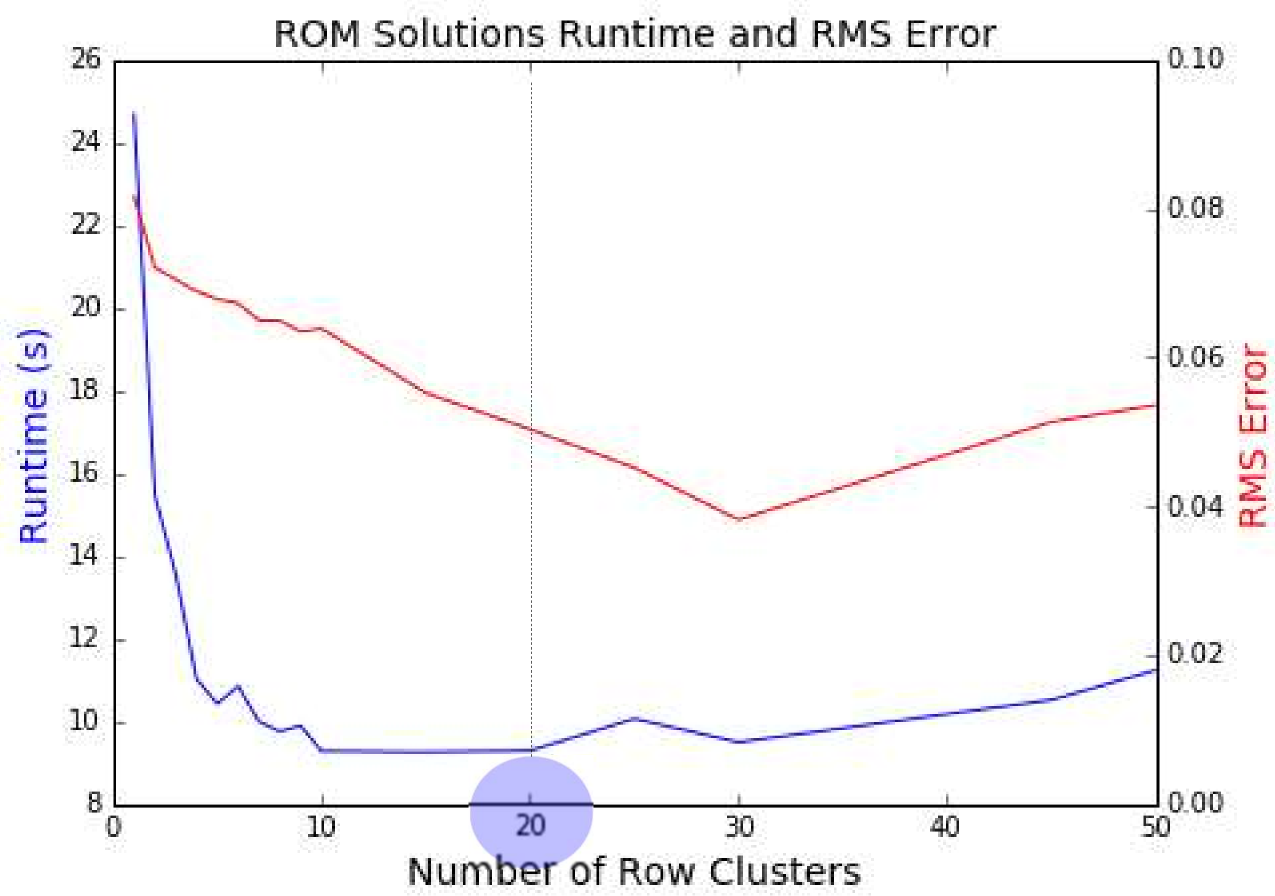


99.9996% energy content

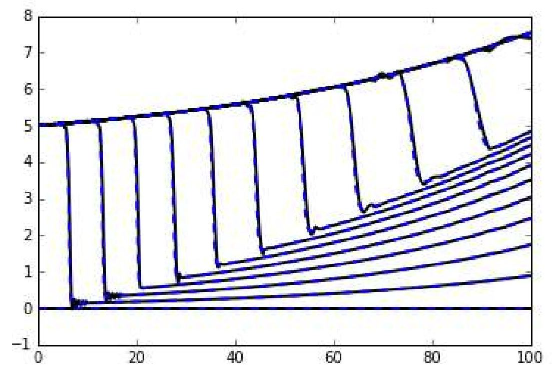


SPATIAL/ROW CLUSTERING ONLY

- Scenario defined by truncation of singular values at 99.9996 %



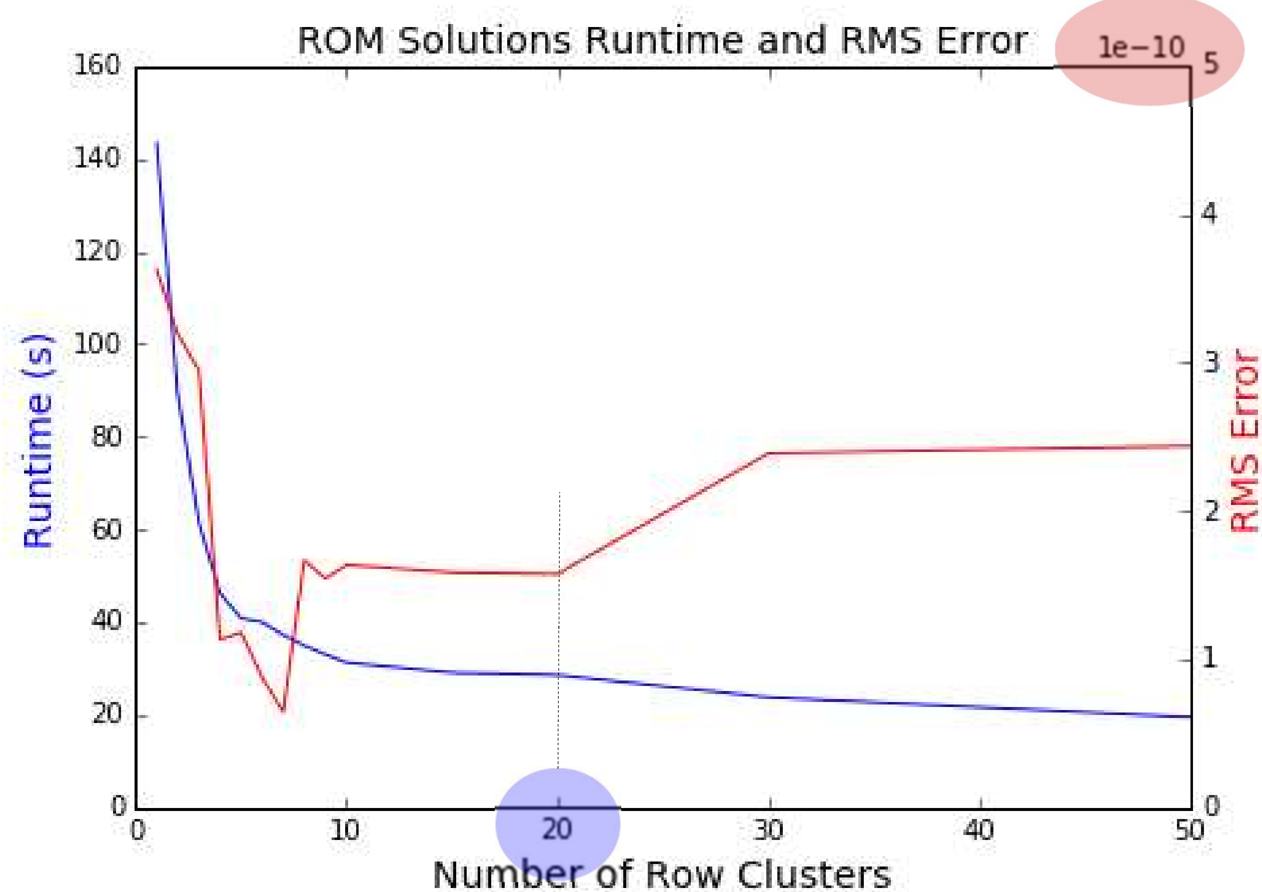
20 row clusters

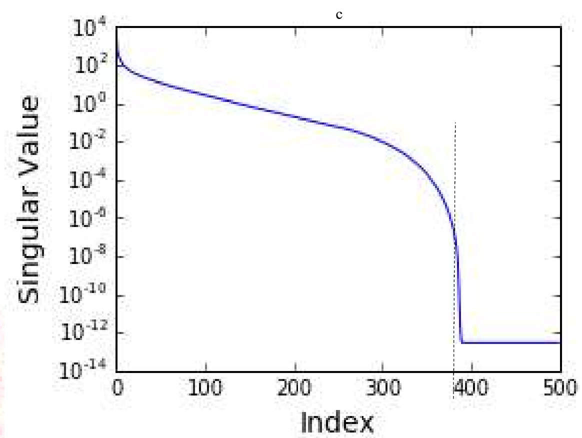


reduction of online CPU time by
a factor ~3 without a significant loss of accuracy

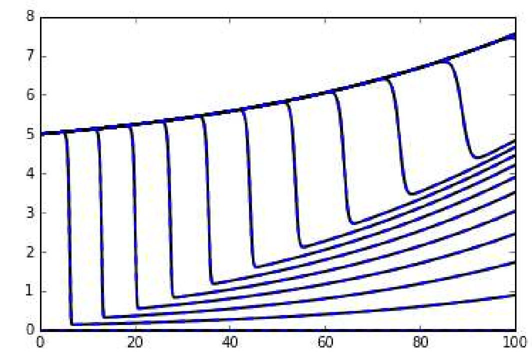
SPATIAL/ROW CLUSTERING ONLY

- Scenario with truncation at ~380 singular values (99.999…99 %)





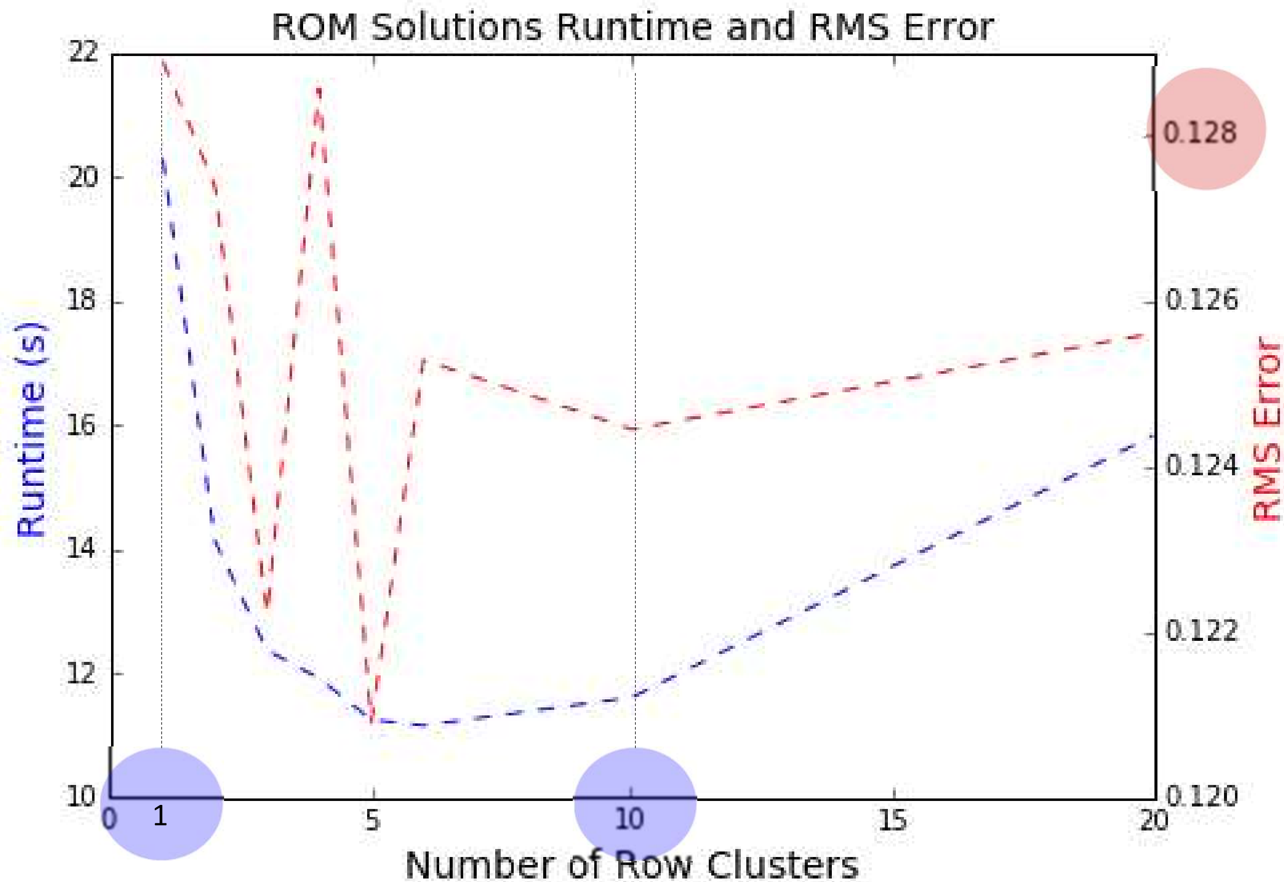
20 row clusters



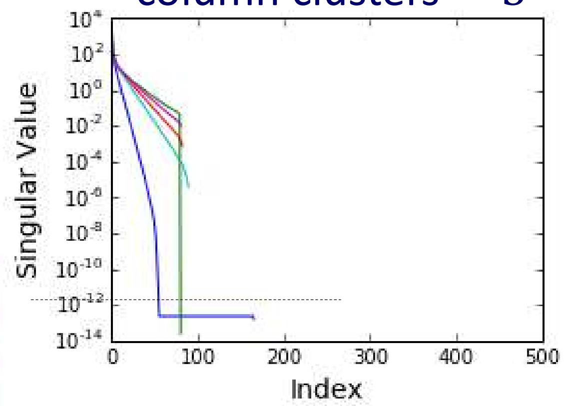
reduction of online CPU time by
a factor 7 without a significant loss of accuracy

ROW-COLUMN CLUSTERING

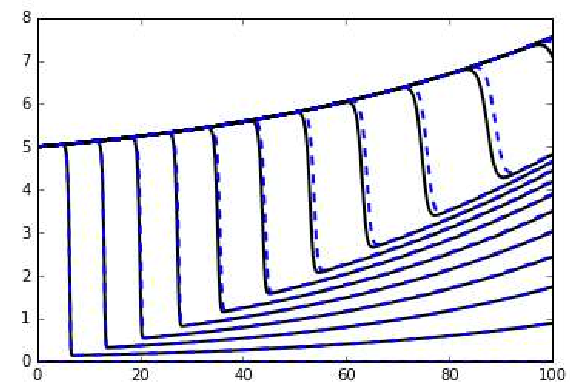
- Scenario with number of column clusters = 5 at 99.99999999996 %



column clusters = 5



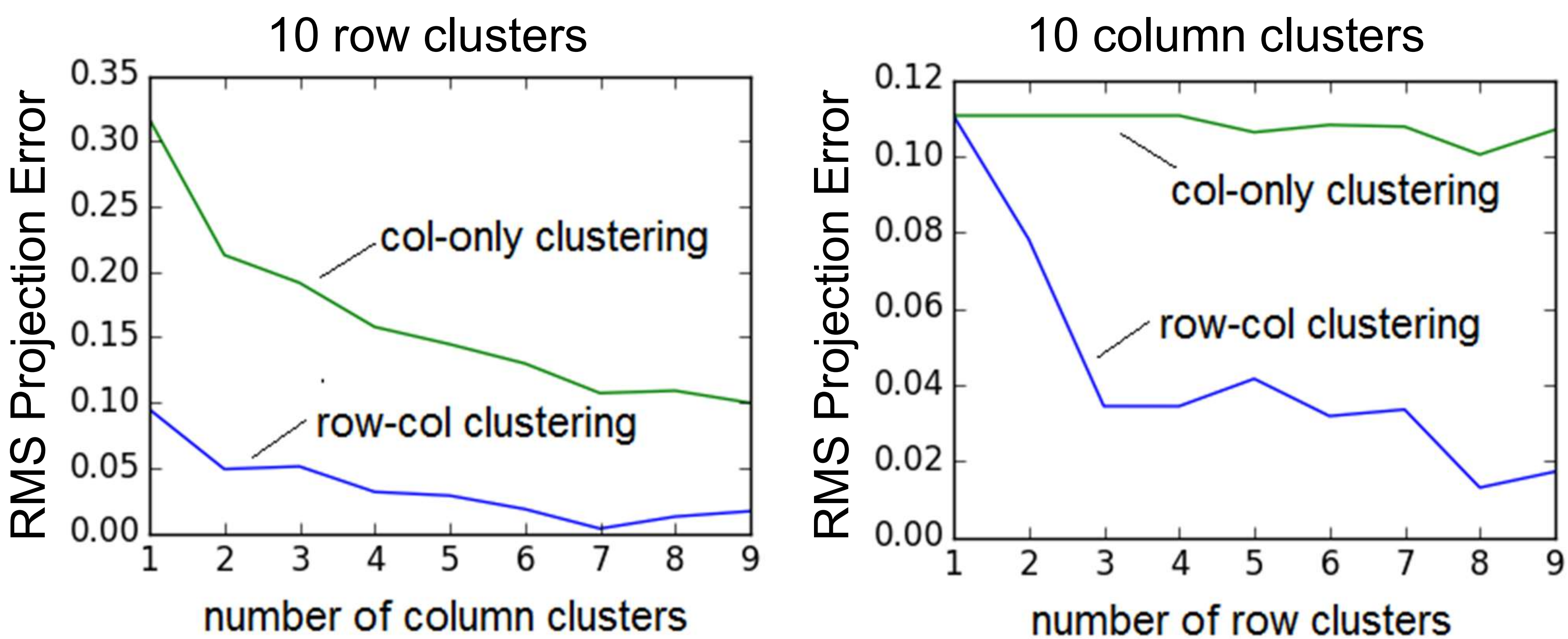
10 row clusters



row clustering outperforms column clustering

ROW-COLUMN CLUSTERING

- Scenario with $n_g = 10$ and roughly the same online cost



→ significant accuracy improvement

CONCLUSIONS

- In the context of local ROBs for nonlinear model reduction, spatial (or “row”) clustering

  - is an effective approach for sparsifying a ROB and therefore accelerating online reduced-order model simulations
  - is a more effective alternative to column clustering for problems with local/localized phenomena
  - can be combined with column clustering to achieve both dimensional reduction and sparsification, and therefore maximize computational eficicency for some problems

- For a simple inviscid Burger problem in 1D and a simple implementation, row clustering has accelerated online nonlinear reduced-order model simulations by a factor ranging between 3 and 7 — larger speedups are expected for 3D problems and an optimized implementation